Rendering Gerrymandering Impotent: A Simple Redistricting Reform*

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Abstract

I introduce a new and novel electoral reform that continues to allow redistricting, but changes the incentives to do so. This reform ensures parties earn seats proportional to their performance at the polls without substantially changing the electoral system in the U.S. In order to evaluate the reform's impacts, I examine a model that incorporates the redistricting decision, candidate choice, state legislative elections, and policy choice. Unsurprisingly, strategic redistricting biases policy in favor of the redistricting party. In the environments studied, the new reform never increases policy bias, and often reduces it.

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1 Introduction

In the United States, electoral boundaries are usually drawn by elected representitives. This "gerrymandering" process allows politicians to choose their voters and causes other conflicts of interest. How should electoral institutions be designed in the face of gerrymandering? In this paper I examine a new, simple reform which ameliorates the effects of gerrymandering. Unlike many such

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reforms, this one would not restrict partisans' abilities to draw electoral districts, but instead reduces the potential impact of changing district boundaries. The reform is simple in a sense: it leaves electoral institutions as they are before the polls close, but alters how winners are chosen. The reform is inspired by proportional representation systems, but essentially maintains single member districts. Under the reform, parties could no longer just find cleaver ways to get the most candidates with 50% of the vote. They would have to get the most votes.

The purpose of this paper is to propose and evaluate the new reform. First, I formally define the reform. Then, I develop a model to examine gerrymandering under the current system and the impact of implementing the new reform. I show that if the reform were adopted, redistricters' ability to bias[1] policy would generally be reduced, and never increased.

The literature on socially optimal redistricting is surprisingly sparse. Many papers, such as Gilligan and Matsusaka (1999), Friedman and Holden (2008), and Gul and Pesendorfer (2010), focus purely on the question of how parties should redistrict to achieve partisan ends. Other papers like King and Browning (1987), and Cox and Katz (1999) test the predictions of such models. However, a few papers such as Coate and Knight (2007) and Gilligan and Matsusaka (2006) focus on the more policy oriented question: how can we improve on our electoral system in the face of gerrymandering? I continue that vein of research by examining policy bias induced by different electoral institutions.

This paper contributes to two distinct lines of research. I further the literature on strategic redistricting by examining a rich model of gerrymandering, elections and policy choice. This model confirms the standard "pack and crack" intuition[2] from prior literature. I also propose a new electoral reform that deviates minimially from current institutions, but has the power to substantially limit gerrymandering. The proposed reform also adds to a large literature on alternative electoral and voting systems. Similar to Single Transferable Vote or Instant Run-Off systems, my reform could be thought of as reducing "wasted votes" that would otherwise exist in districts in which elections are currently a forgone conclusion.

The model I develop yields results that are standard in the literature on strategic redistricting. Under the current system, partisans in control of the redistricting process can substantially bias policy towards their favored position. Also, the process they use is the same as the standard "pack and crack" result found in most of the strategic redistricting literature. After implementing the reform, redistricters may still be able to bias policy in their favor. However, the reform *never* increases that ability and often greatly reduces it.

The rest of this paper proceeds as follows: Section 2 introduces and formally defines my proposed reform. Section 3 lays out the model I will use to examine

[1] Throughout this paper, when used as a noun, (policy) bias will refer to the distance between actual policy and the policy prefered by the median voter. When used as a verb, bias will refer to moving policy towards one's prefered position and away from that of the median voter.

[2] This intuition will be explained later.

gerrymandering under the current system and under my proposed reform. Section 4 provides the solutions to the model under the current system and under my proposed reform. Section 5 discusses the implications of my results and Section 6 concludes.

2 A New Reform

In this section I discuss a new electoral system that in a simple model would completely eliminates the effects of gerrymandering. This reform offers two main benefits. First, it ensures that the distribution of Democrats and Republicans in the legislature is proportional to the votes received by each party in the general election. Second, it requires no changes to the way elections or redistricting are currently implemented in any state. In fact, this reform could be easily added on top of other redistricting reforms such as the Voters FIRST Act passed in California in 2008. I further discuss the benefits after explaining the electoral system.

This reform does not change any electoral institutions before the polls close. It only changes how representatives are chosen from the candidates running in individual districts. Suppose we are discussing elections to a state's House of Representatives which has N districts (seats to fill). Suppose that there is one Democrat and one Republican running for every seat (and no third party or independent candidates). Once the election happens call v_{dn} (v_{rn}) the number of votes received by the Democratic (Republican) candidate in district n. Then let $V_d \equiv \sum_{n=1}^{N} v_{dn}$ ($V_r \equiv \sum_{n=1}^{N} v_{rn}$) be the votes cast statewide for Democratic (Republican) candidates. The share of the votes recieved by the Democratic (Republican) candidate in district n is then $s_{dn} = \frac{v_{dn}}{v_{ni}+v_{rn}}$ ($s_{rn} = \frac{v_{rn}}{v_{dn}+v_{rn}}$) and the share of the vote recieved by Democrats (Republicans) statewide is $S_d = \frac{V_d}{V_d+V_r}$ $\left(S_r = \frac{V_r}{V_d+V_r}\right)$.

Under the current system, in each district the candidate receiving the most votes becomes the representative of that district. In other words the Democrat wins the seat for district i if[3]

$$v_{dn} > v_{rn}.$$

Instead, we could choose representatives according to the following mechanism: List the districts by the Democratic win margin, $s_{dn} - s_{rn}$. This way you have contests where the Democrat defeated her opponent the most readily at the top of the list, and contests where the Republican defeated her opponent most readily at the bottom of the list. In the first $S_d \cdot N$ districts on the list, the Democrat becomes the representative, while in the last $S_r \cdot N$, the Republican becomes the representative. This way, the proportion of seats each party holds in the House is equal to the proportion of votes recieved statewide, and the "strongest" Democratic and Republican candidates are the ones that

[3] I ignore the possibility of ties.

Table 1: Districts/Election Results

District	Democrats	Republicans
1	0	200
2	0	200
3	120	80
4	130	70
5	150	50

Table 2: Election Results

District	Democrats	Republicans
5	**150**	**50**
4	**130**	**70**
3	*120*	*80*
2	*0*	*200*
1	*0*	*200*

get elected.[4] Of course, it's likely that $S_d \cdot N$ and $S_r \cdot N$ will not be integers. In that case, just round each to the nearest integer.[5],[6]

To see how this electoral system works and how it would change the incentives to gerrymander, consider the following example. Consider a state made up of 400 Democrats, and 600 Republicans with a state legislature that will have five representatives. Suppose voters are simple; they always vote for their party. Further, say the Democrats were able to gerrymander after the last census such that districts have the makeup shown in Table 1. Since voters are perfectly predictable, the district makeups in the table exactly mirror votes in the election. As you can see, under the current electoral system, Democrats will win 3 out of 5 seats despite receiving only 2 out of every 5 votes statewide. Alternatively, under the proposed system, the Democrats would receive 2 out of 5 seats because they received 2 out of every 5 votes. To decide which two seats would be Democratic, take Table 1 and reorder it as in Table 2. Now the districts that most heavily favored the Democrats are at the top. Since the Democrats are to receive two seats, the "strongest" two Democrats in Table 2 (those in bold) will

[4]Here "strength" refers to the ability to get a larger share of the vote in your local election.

[5]There are many potential ways of dealing with this rounding issue. If we want to support minority voices, another option is to award the problem seat to the party that will be in the minority. However, Coate and Knight (2007) find that the majority party should optimally have a portion of the seats greater than it's proportion of the vote. Given their result you should award the problem seat to the majority party. In the models I use in this paper, the parties only care about whether or not they've earned a majority. Therefore this is only a problem if it's the decisive seat; in which case you should always award the seat to the party winning the most votes. Dealing with integer problems for other seats becomes unimportant.

[6]This method is not as clean if we wish to include third parties. However, since the U.S. already has a stable two party system, keeping minority parties out of the new system hardly seems objectionable. Specifically, this system could be applied to the top two vote getting parties, or we could ignore districts with a strong third party showing.

win their elections. The "strongest" three Republicans (those in italics) will win their elections.

As you can see, under the reform in one of the districts a Republican is elected while receiving fewer votes than her opponent. Under the current system the Democrats control the legislature while receiving fewer votes than Republicans. In this example, any method of chosing representatives must cause at least one of these undesirable results.[7] The choice is then between allowing some representatives to be elected without winning the most votes, or to allow one party to govern without winning the most votes.

In this simple model, gerrymandering is clearly harmful under the current system. It can bias the legislature away from majority rule. The proposed system completely eliminates this bias without doing away with single member districts. Since partisans are all exactly alike, and voters are perfectly predictable, the proposed system completely eliminates any incentive to gerrymander. All that matters for a party is their share of the aggregate vote, and gerrymandering can not affect this. In the rest of the paper I discuss this reform in the context of a richer model of gerrymandering.

2.1 Benefits

The main benefit of this new reform is that it is in many ways simple. The drawing of districts is a very complicated process and this reform addresses gerrymandering without having to deal with how districts are drawn. It simply alters the election to weaken gerrymandering's potential impact. This allows the reform to be implemented on it's own or act as a supplement to other reforms. For instance, California's Voter's FIRST Act took control of the districting process away from the State Legislature and gave it to a citizen's commision. While the intention was clearly to create a commision that will not have the same conflicts of intrest as the legislature, there is nothing to guarantee that the new goup wont draw districts that bias the legislature towards one party or another (intentionally or not). Adding my reform on top of the Voter's FIRST Act could alleiviate that problem.

Also, this reform could ameliorate the wide concern over the "preverse-effects" of creating majority-minority districts (e.g. see Gilligan and Matsusaka (2006)). The mandate to create such districts is designed to ensure that minorities have some representation in the legislature. However consider, purposefully "packing" a large number of Black voters into one district. This will dilute the number of Black voters in other districts. Since Blacks generally prefer Democrats by a wide margin, this will nearly garuantee that a Democrat is elected in the "Black district" but harm the electoral prospects of Democrats in many other districts. This could result in significantly fewer Democrats being elected to the legislature. Since Blacks tend to prefer Democrats, this may end up being detrimental to their interests overall. Under my reform, packing minority voters (and their votes) into a few districts would no longer harm that

[7] This must be true as one party recieved a majority of the votes in a majority of the districts, while the other party got a majority of the votes statewide.

minority's prefered party. Their votes would still be used in determining each parties overall share of the seats.

The reform would not require significant changes to elections as generally held in the United States. Voter's don't need to vote differently. They can still just choose their preferred candidate to represent them come election day.[8] Furthermore, it maintains an often touted virtue of the United State's version representative democracy: each representative is directly responsible to a unique set of constituents.

***Moderates candidates even in unloseable districts.

***Could be implemented unilaterally by California.

***Has flavor of PR, without getting rid of SMD.

2.2 Concerns

The main concern is likely to be that some representives will have been elected while recieving fewer votes than their opponents. While this may strike some as "unfair," by the same logic, having one party conrol the legislature while recieving fewer votes than the opposing party is also unfair. As noted in the example at the begining of this section, any method of chosing representatives from individual elections must admit at least one of these "unfair" outcomes. The choice is then which to allow.

***Of course changing the electoral system may change parties strategies in other ways. Pack to lower turnout.

***Incumbent gerrymander still possible

***Independents/third parties

***May be difficult to implement. Would require constitutional amendments

3 Model

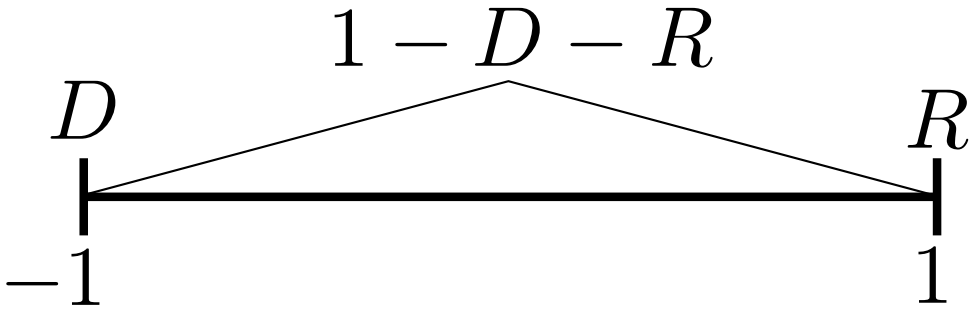
In this paper I consider a four stage model of elections and redistricting. In the first stage, one of the two parties will redistrict with the only consideration being their partisan agenda. In the second, each party will choose a candidate to run in each district. The third stage is the election, where each voter chooses one of the two candidates. Finally, in the fourth stage, the elected representatives gather and implement a policy.

My model captures many of the intricacies of actual gerrymandering. Specifically, the second stage allows me to examine how gerrymandering not only affects the numbers of Democrats and Republicans in office, but also the policy positions of individual candidates and representatives. The third stage allows me to make prediction of the effects of gerrymandering on votes and the final stage provides insight into the impact on policy outcomes.

The ability to to study candidate policy positions is particularly useful. Gerrymandering could have significant effects on the types of representatives

[8] However, it may create an opportunity to vote strategically.

Figure 3.1: A State



elected. Consider the common pack and crack result from the strategic redistricting literature[9] in a state gerrymandered by Democrats. They will pack as many Republican voters as possible into a few heavily Republican districts, while placing more mild Democratic majorities in the remaining districts. Since the Republicans are sure to win their few districts, the Republican primary will essentially decide the election in those districts. Since Republicans need not worry about attracting moderate voters, they are likely to chose a candidate that is extreme. However, since the Democrats have slimmer majorities in the remaining districts, they may still need to worry about losing to a moderate Republican in those districts. As such, they may nominate more moderate Democrats.

In my model, a state has a unit measure of voters. The measures of Democrats, Republicans and Independents statewide are D, R, and $1-D-R$,[10] respectively. Democrats prefer policy -1, Republicans prefer policy 1, and independents prefer policy $x \sim U[-1,1]$. The state can then be represented as in Figure 3.1. The state is to be divided into N Congressional Districts by the party in charge of redistricting.

3.1 Stage One: Redistricting

Without loss of generality assume that the Democratic party is gerrymandering. Their task is then to divide the state's population into N districts of equal population. Here I abstract away from geographic (and other districting) constraints and assume any voter can be placed in any district.[11] However, I also assume

[9] As mentioned, for example, in Gul and Pesendorfer (2010).

[10] Obviously, I assume $D,\ R,\ D+R \in [0,1]$.

[11] On first inspection this may seem like an overly simple assumption. However, consider the requirement that districts be contiguous. Gerrymanderers have been known to create districts with thin portions connecting disjoint regions, effectively bypassing the continuity requirement. (For a famous example, see Illinois current 4th Congressional district.) However, if such constraints are binding one can view the solution to this model as the redistricting scheme gerrymanderers would prefer to implement, absent constraints. Actual redistricting schemes could then be thought of as second best approximations. The result I find regarding gerrymanderers ability to bias policy could then be interpreted as a maximum possible bias.

that the gerrymanderer can only observe party affiliation,[12] but not ideology. With partisans, this distinction is unimportant. However, any independents placed in a district will be a random draw from the overall distribution of independents in the state. Since I assume voters have measure zero, the distribution of independents within a district will be the same as the distribution statewide. Therefore, the gerrymanderer is tasked with choosing proportions of Democrats and Republicans in each district $\{D_n, R_n\} \quad \forall n = 1, ..., N$, respectively. They must respect several constraints. Clearly,

$$D_n,\ R_n,\ D_n + R_n\ \in [0,1] \quad \forall n = 1, ..., N. \tag{3.1}$$

Additionally, all Democrats, Republicans, and independents in a state must be in one of the N equally sized districts. This is equivilant to requiring that the average proporition of any voter type across the districts must equal the statewide measure of that type. Alternatively, we could write this as the following constraint:

$$\sum_{n=1}^{N} \frac{D_n}{N} = D, \quad \sum_{n=1}^{N} \frac{R_n}{N} = R. \tag{3.2}$$

The districting scheme chosen by the Democratic party may impact the policy that is eventually implimented. We will use g to denote that policy. Throughout this model I assume that utility decreases linearly in the distance of the actual policy from one's prefered policy. Therefore, the objective of the Democratic party in this stage is to minimize the expected distance between -1 and the policy which will eventually be implemented, g. As will be apparent later, policy outcomes must be in $[-1, 1]$ so this is equivalent to simply minimizing the expectation of g or:

$$\min_{\{D_n, R_n\}_{n=1}^{N}} E(g). \tag{3.3}$$

The equilibria I find are in pure strategies. So, one can ignore the expectation.

3.2 Stage Two: Candidate Choice

Assume that first the smaller party chooses their candidates' policy positions followed by the larger party. Specifically, if $D \geq R$, first the Republican Party chooses its candidates' policy positions, $\{r_n\}_{n=1}^{N}$ followed by the Democratic party choosing its candidates' policy positions $\{d_n\}_{n=1}^{N}$.[13] If $R > D$ this order

[12] This may be a particularly appropriate assumption in states where voters register by party.

[13] Having the parties play sequentially is unimportant when examining the model under the current system. Under the reform, in equilibrium, the Democratic vote share may be the same accross many districts. This assumption removes the ability for Republicans to randomly choose weaker candidates in a certain district in the hopes of making the Democrat from that district a representative while causing a Republican from another district to become a representative. Without this assumption it does not seem that the game would be solvable in pure strategies under the reform.

is reversed. Without some additional assumptions, this model would essentially collapse to the median voter model. However, the parties are subject to the following constraints:

$$d_n \in [-1, 0] \tag{3.4}$$

$$r_n \in [0, 1]. \tag{3.5}$$

While these constraints may seem arbitrary, they are grounded in reality. In strongly Democratic or Republican districts we rarely see credible challengers. For example, in heavily Republican districts we often see quite conservative Republican candidates. However, we don't generally see only slightly less conservative Democratic challengers that do nearly as well. ******May need to find a citation for this. Probably in my dataset for other paper...****** For whatever reason, the Democrats chose not to, or are unable to, field very conservative (and successful) candidates.[14]

Let $F_d(d_n)$ represent the probability that d_n will win taking the Republicans' strategy as given. Democrats attempt to maximize the expected partisanship of their representative. Specifically, the Democrats' objective function is:

$$\min_{d_n} \left[F_d\left(d_n\right)\left(d_n - (-1)\right) + \left(1 - F_d\left(d_n\right)\right)\left(r_n - (-1)\right)\right], \tag{3.6}$$

If we define $F_r(r_n)$ similarly as the probability that r_n will win taking the Democrats' strategy as given, then the Republicans' optimization problem will be the following:

$$\min_{r_n} \left[F_r\left(r_n\right)\left(1 - r_n\right) + \left(1 - F_r\left(r_n\right)\left(1 - d_n\right)\right)\right]. \tag{3.7}$$

3.3 Stage Three: The Election

Assume district n is made up of D_n, R_n, and $1 - D_n - R_n$, Democrats, Republicans, and independents, respectively, and that Democrats and Republicans have nominated candidates at positions d_n and r_n, respectively. Voters always vote for the candidate closest to them.[15] Therefore, an independent voter at x votes for the Democratic candidate with position d_n if

$$|x - d_n| < |r_n - x|, \tag{3.8}$$

and votes for the Republican candidate with position r_n if

$$|x - d_n| > |r_n - x|. \tag{3.9}$$

I will later discuss tie breaking rules if it matters that for voter x, $|x - d_n| = |r_n - x|$. Assume partisans always vote for their party's candidate.[16]

[14] Perhaps such candidates have trouble amassing the support needed to make it through the party's primary, or are assumed disingenuous as conservative Democrats.

[15] This assumes away the potential for strategic voting under the new reform. Specifically, voters never vote for the candidate farther away from them in order to reduce the aggregate vote share of the party to whose candidate is closer. Strategic voting would likely be mostly against the party which was biasing policy away from the median. As such, admitting this possibility should only strengthen my results regarding policy outcomes.

[16] This assumption is only necessary when both candidates are located at zero.

3.3.1 Current Electoral System

Since there is no uncertainty, the median voter will be decisive if we use the current rules for chosing winners in elections. As long as both parties' make up less than $\frac{1}{2}$ of the district ($D_n, R_n < \frac{1}{2}$) the median voter will be $\frac{R_n - D_n}{1 - D_n - R_n}$.[17] Otherwise, $D_n \geq \frac{1}{2}$ or $R_n \geq \frac{1}{2}$, and the median voter will be the corresponding partisan. (Unless of course $D_n = R_n = \frac{1}{2}$, then let $m_n = 0$, even though there are no independents.) Formally, define the median voter in district n, as follows:

$$m_n = \begin{cases} \frac{R_n - D_n}{1 - D_n - R_n} & if\ D_n, R_n < \frac{1}{2} \\ -1 & if\ D_n \geq \frac{1}{2} > R_n \\ 1 & if\ R_n \geq \frac{1}{2} > D_n \\ 0 & if\ R_n = D_n = \frac{1}{2} \end{cases}. \tag{3.10}$$

The elected representive of district n will then be the candidate chosen by the median voter according to equations 3.8 and 3.9. [18] If the median voter is indifferent between the candidates ($|m_n - d_n| = |m_n - r_n|$), suppose that the candidate of the party with a larger population in that district wins.[19] If the median voter is indifferent and both parties have the same population in the district, elections are decided by the flip of a fair coin.[20]

In order to formally define the policy position of the representative from district n, first define q_n as the difference between how far median voter is from the democratic candidate and how far she is from the republican candidate. Formally,

$$q_n = |m_n - d_n| - |m_n - r_n|. \tag{3.11}$$

This way, $q_n$is less than zero when the median voter is closer to the Democrat and greater than zero when closer to the Republican. Therefore, the representative from district n will be

$$y_n = \begin{cases} d_n & if\ (q_n < 0)\ or\ (q_n = 0\ and\ D_n > R_n) \\ r_n & if\ (q_n > 0)\ or\ (q_n = 0\ and\ D_n < R_n) \\ d_n & if\ q_n = 0\ and\ D_n = R_n \quad (with\ probability\ \frac{1}{2}) \\ r_n & if\ q_n = 0\ and\ D_n = R_n \quad (with\ probability\ \frac{1}{2}) \end{cases} \tag{3.12}$$

[17] The median voter is the policy position with half the voters to the right and half to the left. Using the later we can set up the equation $D_n + \frac{m_n + 1}{2}(1 - D_n - R_n) = \frac{1}{2}$ and solve to find the m_n in the first line of equation 3.10

[18] Technically, there is one more case. If $D_n = R_n = \frac{1}{2}$ then $m_n = 0$, even though there are no independents. To avoid discontinuity problems, assume that in this case the winner of the election is the candidate closer to m_n and that ties are decided by the flip of a fair coin.

[19] One can think of this tying rule as an approximation of the following in an environment where ties are decided by the flip of a coin: In equilibrium, the candidate of the party with a smaller population will be at zero. So, the advantaged party could find the policy position which makes the median voter indifferent, and then run a candidate marginally closer to 0 in order to ensure victory.

[20] In this case the $m_n = 0$ Then, in equilibrium $d_n = r_n = 0$. So, the different realizations of the the tie breaking rule wil not change policy as both candidates are the same.

3.3.2 Reformed Electoral System

The fraction of the vote recieved by the Democratic candidate in district n will equal the number of of Democrats in the district plus the portion of the independent voters who are to the left of the median voter. Using our notation from above,

$$s_{dn} = D_n + \frac{1 + m_n}{2}(1 - D_n - R_n). \tag{3.13}$$

Without loss of generality assume that the distrcits are ordered such that

$$s_{dn} \geq s_{dn+1} \; \forall n = 1, ..., N - 1. \tag{3.14}$$

As the districts are all the same size, the share of the statewide vote recieved by democratic candidates will equal the average share across the districts,

$$S_d = \sum_{n=1}^{N} \frac{s_{dn}}{N}. \tag{3.15}$$

If $D \geq R$, assume the Democratic party will recieve $P_d = \lfloor S_d N + 0.5 \rfloor$ seats in the state House. Otherwise, $D < R$ and assume that the Democratic party will recieve $P_d = \lceil S_d N - 0.5 \rceil$ seats in the state House. This way, the larger party will control the state House so long as they recieve at least 50% of the vote (statewide). The Democratic candidates from the P_d districts with the largest s_{dn} (the first P_d districts) will become representatives. The Republican candidates from the $N - P_d$ districts with the smallest s_{dn} (the last $N - P_d$ districts) will become representatives.

The above description will uniquely determine the representatives from each district unless the margin of victory is exactly the same in districts P_d and $P_d + 1$ (if $s_{dP_d} = s_{dP_{d+1}}$).[21] In this case assume that the larger party[22] gets to pick which of the candidates from the districts in question shall fill their allotted seats.[23] Alternatively, one could think of the larger party choosing the ordering of the $\{s_{dn}\}_{n=1}^{N}$ subject to the constraint that $s_{dn} \geq s_{dn+1} \; \forall n = 1, ..., N - 1$.

3.4 Stage Four: Policy Implementation

Here I draw on Gilligan and Matsusaka (2006) and assume that in stage four, when the legislature meets, it will adopt the policies of the median representative.[24] Therefore, the goal of the Democratic party is to minimize the policy position of the median legislator. For convenience, assume there are an

[21] If the reform were enacted the chances of this happening in the real world would be remote.

[22] the Democratic party when $D = R$.

[23] This is similar to having the redistricting party make small changes in the districting scheme and their choice of candidates such that their prefered candidates go to congress. Obviously, a different tying rule would be a part of any actual reform.

[24] I could provide a larger median voter model for this stage. However, this Downsian reasoning is sufficiently well known to skip directly to the result.

odd number of seats in the legislature. If we reorder the districts such that $y_n \leq y_{n+1}\ \forall n = 1, 2, ..., N-1$, then the policy implemented will be:

$$g = y_{\frac{N+1}{2}}. \tag{3.16}$$

4 Results

Under both the current system and the reform, the game outlined above is a two player zero-sum game.[25] Such games feature the well known constant payoffs property: while such games may have multiple equilibrum, the expected payoffs must be the same in *all* equilibrium. Since payoffs are linear in g, the expected policy outcome must be the same across all equilibria. Therefore, all results relating to policy outcomes are robust to concerns about alternate equilibria.

4.1 Current System

The Nash equilibrium to this model under the current electoral system is a variation on the standard pack and crack result common in the literature. I will first describe the equilibrium, then provide the formal strategies, and and finally show why neither party would deviate.

The Democrats goal is to get $\frac{N+1}{2}$ representatives who are as liberal as possible as measured by the least liberal among them.[26] Therefore, they will create $\frac{N+1}{2}$ "Democratic" districts. In effect this gives them $\frac{N-1}{2}$ "Republican" districts in which to place voters, without regard to the outcome in those districts. In order to achieve their goals they need to move the median voter as far to the left as possible in the "Democratic" districts (since the representative from each district will be $2m_n$ or a the corresponding partisan if $m_n \notin [-0.5,\ 0.5]$. In my model, placing an extra Republican in a district will always move the median voter to the right more than placing an independent (or a Democrat) ***Talk about derivatives...***. Therefore, to maximize the partisanship of the "Democratic" districts they will contain as few Republicans as possible. Alternatively, as many Republicans as possible will be placed in the "Republican" districts. If there is not a sufficient population of Republicans to fill all $\frac{N-1}{2}$ "Republican" districts, as many independents as possible will be placed in the "Republican" districts. This is because Democrats will tend to move the median voter to the left in the "Democratic" districts, while indepents would moderate those district. If there are not enough Republicans and idependents to fill the "Republican" districts, obviously, there will only be Democrats left. They will make up the remaining population of the "Republican" districts, and the "Democratic" districts will only be populated by Democrats.

After filling the "Republican" districts according to the rules described above, the remaining population should be spread uniformly throughout the $\frac{N+1}{2}$ "Demo-

[25] The Democratic Party is attempting to minimize g while the Republican Party is attempting to maximize it.

[26] since the median representative will be the $\left(\frac{N+1}{2}\right)^{th}$ most liberal representative

cratic" districts. Spreading the population non-uniformly would (weakly) decrease the m_n in some "Democratic" districts and (weakly) raise the m_n in other "Democratic" districts (relative to spreading the population uniformly). Since the policy implemented depends (weakly) monotonically on the highest m_n in the "Democratic" districts spreading, spreading the population non-uniformly could not be payoff improving. In each district parties will choose the following candidates.

$$d_n = \begin{cases} 0 & m_n > 0 \\ 2m_n & m_n \in [0, -\frac{1}{2}] \\ -1 & else \end{cases} \tag{4.1}$$

$$r_n = \begin{cases} 0 & m_n < 0 \\ 2m_n & m_n \in [0, \frac{1}{2}] \\ 1 & else \end{cases} \tag{4.2}$$

Thus, the median voter in each district is halfway between the two candidates, or the district is so extreme that a pure partisan[27] will win.

To ensure this is a Nash equilibrium, consider each player's possible deviations. In each district, the party that wins the election is nominating a candidate who is as partisan as possible, but still capible of winning the election. The other party's candidate is at 0, and will lose no matter what they do. So, there are no payoff improving deviations possible in the second stage. The Democrats are optimally gerrymandering by the explination above.

4.2 Reformed

How we think this reform will affect election depends on whether or not we think it will affect stage two: candidate choice. While in Section 4.2.2 I allow parties to be completely strategic, in Section 4.2.1 I first examine the effect of the reform under the assumption that it would not change the relationship between district demographics and the candidates chosen by the parties.

4.2.1 Naive Candidate Choice

Here, suppose that stage two happens exactly the same way as it would have without the reform. Specifically, in each district suppose candidate choice still occurs according to equations 4.1 and 4.2 that we found in Section 4.1. While it may seem unreasonable to assume that the reform would not change the nominating process at all, it may be useful to consider as an extreme alternative to the case where the state parties strategically choose candidates in all districts, ignoring any district specific considerations. While I have not solved the game in this case, the next proposition follows directly.

Proposition 1. *If the reform does not change the candidate choice rules (equations 4.1 and 4.2) which prevail under the current system, the policy outcome*

[27] a candidate located at -1 or 1.

will be weakly less biased towards the gerrymandering party than under the current system.

Proof. The Democrats goal is to elect $\frac{N+1}{2}$ representatives who are as liberal as possible[28] and the districting from Section 4.1 yeilds $\frac{N+1}{2}$ districts where the more liberal candidate is as liberal as possible given equation 4.1. Under the current system the more liberal candidate would win all those elections. However, under the reform that may not be the case. The Democrats may not have a majority of the aggregate vote. Therefore, the policy outcome can be no more biased toward the gerrymandering party than under the current system. □

4.2.2 Strategic Candidate Choice

Now, assume that both parties treat stage two strategically. Specifically, in stage two, both parties take the districting scheme as given and choose candidates across all districts. Recall that the Republicans choose their candidates first, and then the Democrats. If the population of Republicans is greater than that of the Democrats ($R > D$) this will completely eliminate the Democratic party's ability to bias policy away from the median voter (in their own favor). The larger Republican party could run a candidate in each district at the statewide median ($m = \frac{R-D}{1-R-D} > 0$). This would ensure they receive more than half the vote statewide.

In fact the Republicans can do even better than this. In equilibrium the Democrats will make the districts uniform ($D_n = D,\ R_n = R \quad \forall n$). Therefore, $m_n = m\ \forall n$. In order to minimize the ability of the Republicans to bias policy in their favor, the Democrats will choose to run their candidates at $d_n = 0\ \forall n$. The Republicans will then run all their candidates at $r_n = \min\{2m_n, 1\}\ \forall n$.

Instead, consider the case where $D > R$. It is not immediately clear how parties' strategies will change. As we might suspect, the Republicans will want to make it as difficult as possible for the Democrats to win a majority of the seats. Under the reform, the best way to do that is to try and capture as many votes as possible statewide. As such the Republicans will set $r_n = 0, \quad \forall n$.

As before, the Democrats need to create $\frac{N+1}{2}$ "Democratic" districts and attempt to maximize the partisanship of those representatives. At the same time they must ensure that their candidates receive at least half the statewide vote. This will leave $\frac{N-1}{2}$ "Republican" districts. Since aggregate vote is now important the Democrats should maximize votes received in the "Republican" districts. They can do this by setting $d_n = 0$ in these districts.

Since Democrats will be running candidates biased towards the left in the "Democratic" districts, independents in those districts will vote for Democrats less than half the time. However, in the "Republican" districts both candidates are at the median and receive half the independent vote. Therefore, in order to maximize aggregate vote, as many independents as possible will be placed in the "Republican" districts.

[28] Again, as measured by the least liberal among them.

After first placing independents in the "Republican" districts, there are two possibilities. One is that there are no left over independents that need to be placed in the "Democratic" districts. In this case there are only Democrats and Republicans left. If the "Republican" districts are not yet full, the districts will be filled with Republicans. The remaining population will be divided evenly among the Democratic districts. In this case more than 50% of every Democratic district will be Democrats, and the rest will be Republicans. In the "Democratic" districts $d_n = -1 \quad r_n = 0$. The Democrats will have successfully gerrymandered the median legislator to their prefered policy position. However, since in this case it was possible to get $\frac{n+1}{2}$ districts which are more than 50% Democratic, this is the same policy outcome we would have found without the reform.

The second possibility is that there are too many independents to place in just the "Republican" districts. In this case, the "Republican" districts will actually only be populated by independents, and the rest of the population will be spread evenly across the "Democratic" districts. Since the two parties are splitting the "Republican" districts 50/50, the Democrats will need to ensure that they receive at least 50% of the vote in the "Democratic" districts as well. Therefore in these districts they will set $d_n = 2m_n$.[29]

Under the current system, I predict Democrats should gerrymander by first pushing Republicans out of the "Democratic" districts, and then by pushing out independents. Under the reform that order is reversed. This ensures that there are "weakly" more Republicans in the "Democratic" districts under the reform. This implies that in those districts, m_n is greater (less biased) than under the current system. Therefore, the median legislator is less biased under the reform.

4.2.3 Numerical Examples

In this section I consider a few numerical examples

5 Discussion

The electoral mechanism I propose achieves proportionality. However, unlike normal proportional represetation systems, it doesn't sacrifice an often touted feature of single member districts: all representives are directly answerable to a unique constituancy back home.

6 Conclusion

My paper's main contribution is the new redistricting reform. I develop a model which demonstrates the reform's benefits and limitations. With or without the reform, a clever redistricter can sometimes bias policy away from majority rule. However, the reform often shrinks the redistricter's ability to bias, and never increases it. Furthermore, the reform seems to completely change the optimal

[29] Unless of course $2m_n < -1$, in which case $d_n = -1$.

gerrymandering technique. Under the current system the redistricter wants to pack their opponents into a few districts and leave the rest filled with independents and members of her own party. The reform encourages redistricters to instead segregate independents into a few moderate districts, while maintaining a marjority in other, highly partisan districts. This result would seem to make districts more competitive, and adding uncertainty to this model should make gerrymandering more difficult.

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